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THE AGE AND AFFINITIES OF THE TERTIARY FLORA OF WESTERN CANADA

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In 1922 I prepared an account of fossil plants from various localities in British Columbia and Saskatchewan for the Geological Survey of Canada. Publication of this report has been delayed, and it seems desirable to record certain of its more important conclusions without going into the details upon which these rest. These conclusions relate to the age, affinities, and environment indicated.

Beginning with the work of Oswald Heer in 1868 on fossil plants brought back from the mouth of the Mackensie River by Richardson and those collected in Alaska by Russian explorers, these floras were regarded as Miocene in age, and Heer's opinion was followed by Sir William Dawson and Professor Penhallow—the two subsequent contributors to our knowledge of the fossil floras of western Canada. Meanwhile, with the transfer of Alaska to American sovereignty and its active exploration by the United States Geological Survey, large collections of fossil plants were accumulated in Washington, largely from what came to be known as the Kenai formation. These have been studied for the most part by Knowlton and Hollick, although the final report of them has not yet been published.

Of late years it has been rather generally recognized that these so-called Kenai floras, as well as Heer's Arctic Miocene, widely distributed in the Arctic Archipelago, was much older than Miocene, and was really upper Eocene or Oligocene in age, or possibly represented both of these horizons. The proof of this is ample and need not be repeated in the present connection. The present study is in entire harmony with this conclusion, since none of the Tertiary which I studied is as young as Miocene, and it all fits our present conceptions of a late Eocene flora.

Only a single qualification of this statement is necessary. Two different floral horizons, separated by some 4000 feet of strata, are recognizable, and it may be possible that both late Eocene and Oligocene are represented by plants in western Canada. The impossibility of reaching a decision at the

present time is due to a lack of knowledge of North American Oligocene floras outside of those of a sub-tropical character in the far southeastern part of the continent which throw no light on temperate Oligocene floras.

Many incorrect opinions have been published regarding the Tertiary climates of the far north and their causes, and these range through varying degrees of improbability all the way to that which would ascribe them to a wandering pole. There is a quite general impression among geologists and botanists that these arctic and sub-arctic Tertiary floras were tropical in character. This is not true of any of them, and that of western Canada is overwhelmingly temperate in its facies.

The bearing upon geographical origin and subsequent dispersal of these Canadian Eogene floras is even more interesting than their age and ecology. In the first place they are decidedly different from the Eogene floras of southeastern North America, which last are essentially of equatorial American origin. The two or three common elements are probably polymorphic forms of a once Holarctic range such as Taxodium, Glyptostrobus, Sequoia and Lygodium, which might have spread in either direction and which are without especial significance.

The Eogene floras of western Canada are typical hardwood assemblages of broad leaf types mixed with coniferous trees, and in many features they show an astonishing resemblance to existing species of the forested upland region of China, which region, with southeastern North America, have been quite properly regarded as vast reserves of Tertiary survivors.

These Eogene floras of Canada show the following categories of relationships: 1. Genera still represented in eastern Asia and southeastern North America by closely related species, as in Sassafras, Liriodendron and Hicoria. 2. Genera of previous Holarctic range now surviving in China, such as Ginkgo and Glyptostrobus; or in Pacific North America, such as Sequoia; or in southeastern North America, such as Taxodium. 3. Genera surviving only in eastern Asia such as Corylopsis and Trochodendron. 4. Genera surviving in both Asia and North America in which the Eogene species are about equally divided in their affinities to existing species in the two regions. This is especially marked in Alnus, Carpinus and Corylus.

I conclude that the two main elements in the Eogene floras of western Canada are an older part, both gymnospermous and dicotyledonous, representing survivors in a region of heavy rainfall and hence with a mild and equable climate, which may have invaded the region from lower latitudes in either North America or Asia, or may have simply become restricted in this region by reason of environmental changes in the western United States or elsewhere; and a second and major element, made up of newer types which entered North America from Asia during the Upper Cretaceous or the earlier Eocene by way of land connections in the Bering Sea region.

It is this second element that has exercised such a marked influence on the subsequent Tertiary and existing floras of temperate North America, and which is largely responsible for the parallelism between the existing floras of North America and Asia, which was first emphasized by Asa Gray so many years ago.

THE METASTABLE $2p_3$-STATE OF MERCURY ATOMS

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1. Let us consider a mixture of mercury vapor, thallium vapor and nitrogen in a closed quartz tube under well defined conditions of pressure and temperature, illuminated by λ 2537Å of constant intensity.

Due to the absorption of the illuminating radiation a constant number N of mercury atoms will be activated in unit time. Those activated atoms persist in the $2p_2$-state for a finite time t which is known to be of the order of magnitude of 10^{-7} sec.[1] If they escape collision during this period they return to their normal state in the ordinary way—emitting radiation (resonance). They may, however, while activated, collide with nitrogen molecules, normal mercury atoms or normal thallium atoms. We shall assume, for the present, that the amount of nitrogen and thallium sufficiently exceeds the amount of mercury, so that we may disregard totally the collisions between activated and normal mercury atoms.

We assume furthermore that all collisions between activated mercury atoms and thallium atoms result in the activation of the latter, producing the fluorescence radiation of thallium with a constant energy distribution between its different lines.

As to the collisions between activated mercury atoms and nitrogen molecules two possibilities have to be taken into account:

The total amount of energy stored up in activated mercury atoms may be used in the collision and converted into kinetic energy of the colliding particles. The activation energy is then divided between them according to the principle of conservation of momentum; the activated mercury atom returns to its normal state without emitting radiation (collisions of the second kind).

It is, however, also conceivable that the result of such a collision does not consist in a direct return of an activated atom to its normal state. The main part of the activation energy may be retained in the metastable $2p_3$-state of the mercury atom, the difference only being converted into

the kinetic form. Such a "partially" activated mercury atom, in the $2p_3$-state, would retain its energy indefinitely in vacuo (selection principle), it will actually retain it during a time t_m sec., determined by the pressure and the nature of the surrounding gas.

The enhancing effect of nitrogen and argon on the indirectly excited fluorescence of thallium vapor *in sealed off tubes under well defined conditions of pressure and temperature*[2] shows conclusively that some of the activated mercury atoms actually persist longer than 10^{-7} sec. in the metastable $2p_3$-state, and while enduring many collisions with nitrogen and argon, lose their energy only in collisions with thallium and with normal mercury atoms. The question arises whether it will be possible to estimate the order of magnitude of the time t_m of the metastable atoms in the $2p_3$-state *from the measurable ratio of the increase of intensity for a given line in the fluorescence spectrum of thallium?*

2. We call N the number of activated atoms produced in unit time by the effective radiation. If t is their life-time in the $2p_2$-state and T the time between two collisions of mercury atoms and nitrogen molecules,[3] then

$$\frac{t}{T+t}$$

measures the fraction of mercury atoms which collide while activated. We know from Stuart's[4] measurements that when activated mercury atoms collide with nitrogen molecules, only 1,3% of the collisions are "collisions of the second kind." If we assume *that all of them result in throwing the colliding mercury atom into the metastable* $2p_3$*-state,* we obtain for the number of metastable atoms produced in unit time:

$$N_m = \frac{1,3}{100}\,\frac{t}{T+t}N. \qquad (1)$$

After a stationary state has been reached there will exist at every moment in our mixture $\mathfrak{N}$ activated mercury atoms (in $2p_2$-state) and $\mathfrak{N}_m$ "partially" activated metastable mercury atoms (in $2p_3$-state). The ratio of those numbers is evidently:

$$\frac{\mathfrak{N}_m}{\mathfrak{N}} = \frac{N_m}{N}\cdot\frac{t_m}{t} = \frac{1,3}{100}\cdot\frac{t_m}{T+t}. \qquad (2)$$

The intensity of the indirectly excited fluorescence spectrum of thallium depends under constant conditions of temperature and partial pressures of the vapors only on the *total* number of activated mercury atoms. This number was $\mathfrak{N}$ as long as no nitrogen was present in the tube; it is $\mathfrak{N} + \mathfrak{N}_m$ when nitrogen is added to the mixture. Due to its presence

the intensity of a given line say λ 5350Å increases α times. The increase of intensity in the ratio α is then equal to the ratio:

$$\frac{\mathfrak{N} + \mathfrak{N}_m}{\mathfrak{N}} = 1 + \frac{1,3}{100} \cdot \frac{t_m}{T + t} = \alpha. \qquad (3)$$

3. An estimate of the order of magnitude of t_m may be obtained from a comparison of the blackening of the photographic plate by a given line, f.i., λ 5350Å of the fluorescence spectrum of thallium, excited without any, or with some, nitrogen present in the tube.

Photographs of this kind were recently published by the author in the *Physical Review*.[5] Some of the original negatives were used for a microphotometric comparison carried out by Miss L. Ware from the staff of the Carnegie Mount Wilson Observatory in Pasadena. The originals were made: (1) on the same plate, one below another, (2) under the same conditions of uniform temperature, (3) with the same time of exposure.

Figure 1 shows the "intensity curves" obtained by means of the registering microphotometer of Koch-Babcock, Pettit and Nicholson.[6] Since the original was a negative the lines are represented by depressions in the curves. Due to reflection and scattering of light from the illuminating arc, all mercury lines appear also on the plate. The thallium lines are indicated by arrows. Each plate contains sections of a curve corresponding to a run through "clear glass" as well as to runs corresponding to "total blackness." These curves help to eliminate the errors due to non-uniformity of the plate itself. The steadiness of the records was tested independently and proved to be trustworthy. Special runs, made through clear glass over the whole length of the plate, coincide exactly with the corresponding sections on the plates used for the measurements. With regard to the approximate character of our estimate of the order of magnitude of t_m no attempt was made to correlate precisely the actual density curve of the negative to the true intensity curve of the spectrum. It is sufficient to regard the ratio α as numerically equal to the ratio of the depressions of the curve corresponding to a given thallium line. The assumptions involved in the derivation of formula (1) allow at any rate only an estimate of the *lower* limit of t_m. The correction, brought in eventually by a precise determination of the relation between the density curve and the intensity distribution (monochromatic intensity marks printed on the same plate!), could evidently affect the result only in one direction, i.e., it would at most increase the value of α.

The curve A gives the density curve of the spectrum excited in a tube containing mercury at 0,5 mm. Hg and thallium at saturation pressure.[7]

The curve B corresponds to the spectrum excited in a tube which con-

tains mercury at 0,16 mm. Hg, thallium at saturation pressure and nitrogen at 5 mm. pressure.

The time of exposure was in both cases 30 minutes. The constant and uniform temperature of the tubes was 590°C. The comparison shows that the depression of the curve at the point corresponding to $\lambda = 5350$Å, is about 9 times deeper in the case when nitrogen is present in the tube. Putting then

$$\alpha = 9; T = 6, 2.10^{-7} \text{ sec.}; t = 10^{-7} \text{ sec.}$$

we get

$$t_m > 3,8 \cdot 10^{-4} \text{ sec.}$$

4. It must be remembered, however, that the time t_m is not a "life-time" of the atom in the same sense as the t in $2p_2$-state is. The time of existence of an atom in the metastable $2p_3$-state will depend on the number of its collisions with other atoms. These collisions are "effective" not only when the second colliding atom is a normal thallium atom but also in the case when it is a normal mercury atom. We must expect therefore that, since the increase of intensity (α) of thallium fluorescence due to the presence of nitrogen appears relatively weaker when the pressure of the mercury vapor in the illuminated tube is very much higher than that of thallium,[8] also the time t_m, computed from the result of a comparison of corresponding spectra will come out smaller. Comparing, for instance, the curve A with a similar curve registering the spectrum excited in a tube containing mercury at 0,5 mm., nitrogen at 8 mm. and thallium at saturation pressure at the same temperature as before, we find

$$\alpha = 2,6; T = 3,8 \cdot 10^{-7} \text{ sec.}$$

and therefore

$$t_m > 0, 66 \cdot 10^{-4} \text{ sec.}$$

The time t_m being not an atomic constant but a quantity determined by the number of "effective" collisions, it may be expected to change with increasing pressure of thallium. The saturation pressure of thallium increases rapidly with increasing temperature. From less than 0,056 mm. at 590°C. it comes up to 0,45 mm. at 680°C. An analogous comparison of the intensity of the green thallium line excited indirectly in the same tubes at a temperature of 680°C. should lead therefore to smaller values of t_m. Corresponding to a ten-fold number of available thallium atoms the "life-time" of the metastable $2p_3$-mercury atoms should come out to be about 10 times shorter, and a computation carried through precisely as above checked this conclusion, i.e., it came out of the order 5.10^{-5} seconds. It must be born in mind, however, that this value is probably too low on

account of the circumstance that the assumption as to the ratio α being an approximate measure of the ratio of intensities will probably not be justified any more under conditions existing at the temperature 680°C.

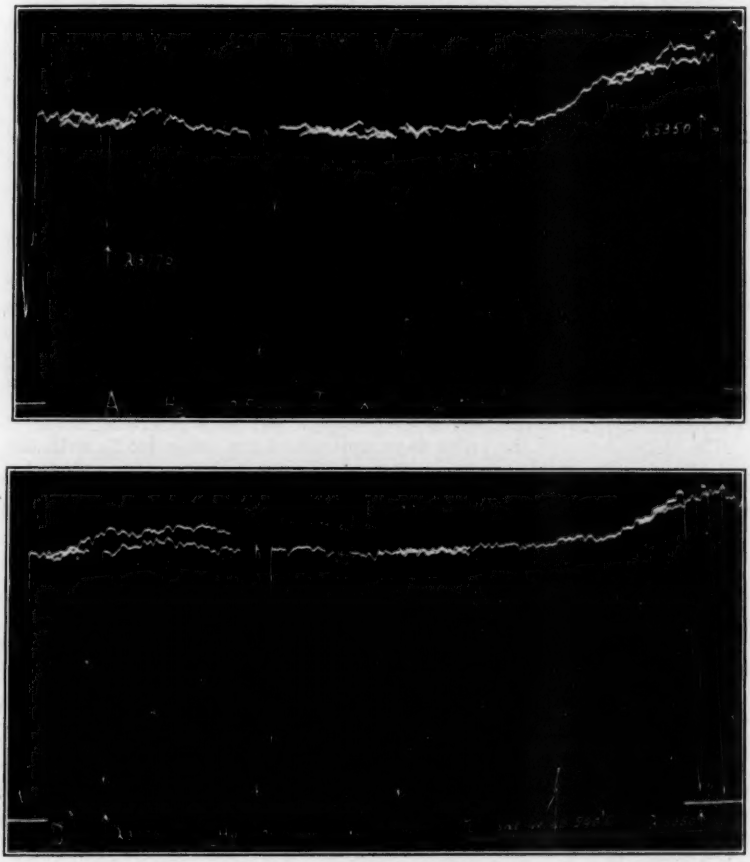

FIGURE 1

Intensity curves obtained from the negatives by means of a registering microphotometer. *A*. No gas present in the tube. *B*. Nitrogen at 5 mm. pressure enhances the intensity of Tl-lines.

5. It is interesting to notice that the order of magnitude of 10^{-4} sec. we obtained above agrees well with the conclusions drawn by H. W. Webb[9] from experiments of an entirely different nature. He produced the metastable state of mercury atoms at low pressures by electron impacts

of 4,9 volts and measured the persistence of the radiation by means of a photo-electric device. The interpretation of his results is based on the assumption "that the excited atoms remain in a metastable state, carry the energy of excitation to the photo-electric surface and there give it up." The lapse of time between the excitation and the arrival at the plate was about from $1{,}3.10^{-4}$ to $2{,}8.10^{-4}$ sec.

Webb finds it difficult to assign the functions of the carriers of the stored-up energy to the atoms in the $2p_3$-state and is inclined to regard rather the atoms in the $2p_2$-state as responsible for the energy transfer. That would mean that also atoms in $2p_2$-state ought to be regarded as metastable under his experimental conditions—a conclusion hard to reconcile with Bohr's theory and with the results of all indirect measurements of t.

On the other hand, the sensitized fluorescence is always accompanied by a strong resonance of mercury vapor. The presence of this resonance shows conclusively that a relatively large number of activated atoms actually escapes collisions and returns undisturbed to the normal state. The time between collisions of mercury atoms with nitrogen molecules being in our cases $3{,}8.10^{-7}$ sec. and $6{,}2.10^{-7}$ sec., this return could have occurred in average after 10^{-7} sec. as we assumed.

The coincidence of the order of magnitude of our value for t_m with the values found by Webb seems therefore to support the view that also in his experiment rather the atoms in $2p_3$-state than those in $2p_2$-state have to be regarded as the metastable ones.

Another attempt to determine the time of existence of the mercury atom in the metastable $2p_3$-state was made by M. Marshall.[10] He used for this purpose the current potential curves for alternating and intermittent arc obtained by use of a Braun tube. The results lead to a value 10 times higher than the lower limit computed above, i.e., about 5.10^{-3} sec. It must be pointed out, however, that recently serious objections have been raised[11] against the interpretation of the results obtained with the method used by Kannenstine and Marshall.

The author wishes to express his best thanks to Miss Louise Ware for the very careful microphotometric examination of the plates and his appreciation to Dr. W. S. Adams, Director of the Mount Wilson Observatory, Dr. E. Pettit and Dr. S. B. Nicholson for the permission to use the facilities of their laboratory for this purpose.

[1] W. Wien, *Ann. d. Phys.*, **73**, 483 (1924); R. C. Tolman, *Phys. Rev.*, **23**, 693 (1924); L. A. Turner, *Ibid.*, **23**, 464 (1924); J. A. Eldridge, *Ibid.*, **24**, 234 (1924).

[2] S. Loria, *Physic Rev.* for November, 1925.

[3] Computed by means of the well known formula: $\nu = 2666{,}6\sigma^2 p \sqrt{\frac{2\pi N}{k\vartheta}} \sqrt{\frac{m+m'}{mm'}}$.

[4] H. Stuart, *Zeit. f. Phys.*, **32**, 262 (1925).

[5] S. Loria, *Physic. Rev.* for November, 1925.

[6] E. Pettit and S. B. Nicholson, *Journ. Opt. Soc. Amer.*, **7**, 195 (1923).

[7] Less than 0.056 mm. This value was obtained by Villiers (*Ann. d. Chem.*, **30**, 558 (1913)) for 634°C.

[8] S. Loria, *Physic. Rev.* for November, 1925.

[9] H. W. Webb, *Physic Rev.*, **24**, 113 (1924).

[10] M. Marshall, *Astroph. Journ.*, **60**, 243 (1924); F. M. Kannenstine, *Astroph. Journ.*, **55**, 345 (1922); **59**, 133 (1924); *Physic. Rev.*, **19**, 540 (1922); **20**, 115 (1922); **23**, 108 (1924).

[11] C. Eckart, *Science*, **61**, No. 1585, 517 (1925).

SOME EXPERIMENTS ON COLLISIONS OF THE SECOND KIND

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The idea of the conversion of kinetic energy into potential energy by a so-called inelastic collision between an electron and an atom had been current for a number of years before Klein and Rosseland[1] showed by thermodynamic reasoning that the reverse process must also occur. According to this principle as extended by Franck,[2] an excited atom, on return to the normal state, may give up its excess energy either as radiation or as kinetic energy shared with an electron or another atom with which it collides. This idea has already proved extraordinarily fruitful and is supported by a considerable amount of experimental evidence.[3,4] The evidence is, however, of a rather indirect sort and it therefore occurred to the writer that an attack on the problem from a different angle might be of interest. Some efforts in this direction are described in the present paper.

The Electron Affinity of Iodine.—An iodine atom has an electron affinity of 3.5 volts. That is to say, an electron has a tendency to attach itself to an iodine atom, thereby forming a negative ion and once the ion is formed 3.5 volts work must be done to remove the electron. Thus, the system formed by an iodine atom and an electron at a great distance from it may be considered as having 3.5 volts of potential energy which must be dissipated in some way during the formation of the negative ion. The possibility of radiation arising from this process has been considered by other investigators;[5] but may not the energy be lost equally well by a collision of the second kind? We know little or nothing of the process of binding of the extra electron, but it seems reasonable to consider it as analogous to the binding of an electron by an ionized atom. If, therefore, one electron comes within the field of a neutral iodine atom and another "collides" with the system so formed before the process of binding the first electron is completed, the second electron may be expected to proceed with increased kinetic energy gained by a collision of the second kind.

The great advantage of an experiment based on these ideas is that the formation of the excited systems involves the presence of no high speed electrons. All that is necessary is a sufficiently high temperature to cause dissociation of the major part of the iodine.

The arrangement of electrodes used is shown in figure 1 where F is an equipotential lime-coated source of electrons, G is a platinum gauze, and P a bucket shaped platinum electrode. This system was mounted in a pyrex glass tube surrounded by an electric furnace and maintained at a high temperature.

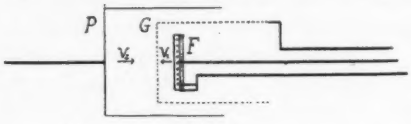

FIGURE 1

A small fixed accelerating field V_1 served to bring electrons into the space between G and P and the number getting to P was measured on a galvanometer as a function of a retarding field V_2. In such an arrangement, if any collisions of the second kind take place, some electrons should be able to get to P even when $V_2 > V_1$. Or, rather, allowing for the failure of the true velocities to correspond to the applied potentials, there should be a fraction of the electrons still getting to P after V_2 has become large enough to cut off the main group.

The curve in figure 2 gives evidence of two such fractions with velocities about 3.0 and 5.0 volts greater than that of the main group. Another curve corroborated this result and a blank run without iodine showed no effect. However, the currents were cut down so by the presence of iodine that the measurements were difficult. Further, the pyrex tube collapsed

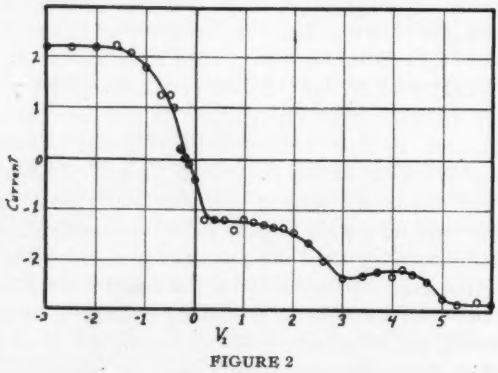

FIGURE 2

after a few runs and attempts to repeat the experiments in quartz met with early disaster. The results, therefore, are far from conclusive but it is hoped that they may be suggestive. Indeed, they have already suggested an extension of the ideas involved to certain phases of chemical reaction.

The Decomposition of Ozone.—According to certain chemical theories, before a molecule can react, it has to become activated by acquiring a certain amount of energy known as the critical increment. For example, in a polyatomic gas which is undergoing thermal dissociation, there must be present normal molecules, activated molecules, and the products of dissociation. Further, the activated molecules do not necessarily break up but may return to the normal state dissipating in some way an amount of energy equal to the critical increment. Suppose this energy may be dis-

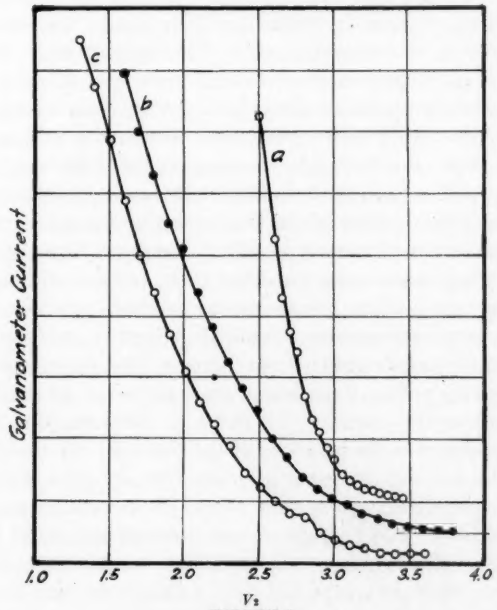

FIGURE 3

Curve *a*. No ozone. Low temperature. $V_1 = 4.00$.
Curve *b*. Ozone. Low temperature. $V_1 = 4.00$.
Curve *c*. Ozone. High temperature. $V_1 = 4.00$.
Curves displaced vertically to avoid overlapping.

sipated either by radiation or by a collision of the second kind, then the latter process might be detected by experiments of the same type as those on iodine already described.

The chief practical difficulty is to find a sufficiently simple chemical reaction. Up to the present the only one tried is the decomposition of ozone but this has been found far from ideal. In the first place the decomposition apparently can proceed in at least three different ways[6] to which correspond three different critical increments of about 12,000, 40,000 and 110,000

calories. In the second place, since at the low pressure used oxygen freezes out in liquid air with the ozone,[7] it is difficult to obtain a simple criterion for the rate of decomposition of the ozone.

Nevertheless a considerable number of experiments have been made and enough results obtained to encourage further work. The same arrangement of electrodes and the same procedure as in iodine were used.

The first experiments were made at room temperature, beginning with a blank run and then with a pressure of about 0.2 mm. of ozone in the tube. The smooth Curves *a* and *b*, figure 3, are typical of results obtained under these conditions. However, when the temperature was raised to 300° curves of the type of *c* were obtained. The small "hump" near the foot might well be considered an observational error were it not present in all curves taken under the same conditions. Altogether twenty runs have been made, three blanks with no ozone but at both low and high temperatures, seven with ozone at room temperature and various pressures and ten at temperatures from 200° to 330° and various pressures. "Kinks" or "humps" appear on none of the blanks and on but one of the low temperature runs, and there very dubiously. On the other hand, all but one of the high temperature runs show the effect fairly distinctly. The one failure was a case of very low thermionic current and consequently inconveniently small galvanometer readings. Further though the observed positions of the "kinks" varied from $V_2 = 0.7$ to $V_2 = 3.00$ when their distances from the points of maximum slope of the curves were determined the variation was only between 1.0 and 1.7. This suggests that the apparent discrepancies in the positions of the "humps" are due to an erratic effect of the ozone and the heating on the contact difference of potential between the electrodes, and do not represent real discrepancies. These limits include both "kinks" from several curves which showed two about 0.3 volt apart. The voltages corresponding to the critical increments given in a preceding paragraph are 0.52, 1.73 and 4.3, but in view of the uncertainty of these values and also of the experimental data just presented, attempts at correlation seem premature.

By substituting an electrometer for the galvanometer and making some modifications in the apparatus, the writer hopes to obtain more definite results and to proceed to the study of other reactions.

[1] *Zs. Physik*, **4**, 46 (1921).

[2] *Ibid.*, **9**, 259 (1922).

[3] J. Franck, *Ergebnisses d. Exakten Naturwissenschaften*, Vol. II, pp. 106–123.

[4] K. T. Compton, *J. Opt. Soc. Amer. and R. S. I.*, **7**, 955 (1923).

[5] See O. Oldenberg, *Zs. Physik*, **25**, 145–149 (1924) for discussion and references.

[6] W. C. McC. Lewis, *Faraday Soc. Trans.*, **17**, 577–580 (1921–22).

[7] Rosenfeld and Schwab, *Naturwissenschaften*, **10**, 470–1 (1922).

GENERAL CRITERION FOR THE CIRCULAR LOCUS OF THE CURRENT VECTOR IN A. C. CIRCUITS AND MACHINERY

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When an electric circuit consists of a constant reactance and a variable resistance, and is connected to a source of constant sinusoidal voltage, the locus of the current vector, under established conditions, is a semi-circle.[1]

This is one of the simplest cases when the current locus, with a constant applied sinusoidal voltage, is a circle. The extremity of the current vector moves on a circle also with some other combinations of resistance, inductance and capacity, as well as in some types of electrical machinery. Thus, disregarding saturation in the magnetic circuit, circular loci obtain for the single-phase and polyphase induction motor, synchronous motor, and some types of a. c. commutator motors. On the other hand, for some other types of machines, the current locus is not a circle, even without saturation. As an example, the single-phase compensated repulsion motor (also known as the Latour-Winter-Eichberg machine) may be cited.[2]

Usually the circular locus is deduced in each particular case from geometric considerations, such as similar triangles, two vectors perpendicular to each other, etc. Or else the method of inversion is used.[3] With the introduction of Vector Analysis into electrical engineering problems,[4] a much more direct and shorter way of finding the position of a diameter of the circle has become possible. Since this method is entirely automatic, it also affords a direct criterion as to whether in a given circuit or machine the locus is a circle or not. It is the purpose of this article to establish such a criterion.

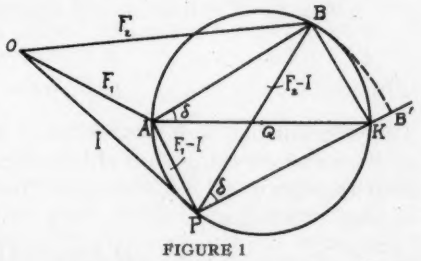

FIGURE 1

Referring to figure 1, let a circle be defined by a chord, AB, and by the angle δ which AB makes with the diameter AK. Let O be the origin and let AB be determined by the known vectors $OA = F_1$ and $OB = F_2$. Let P be a point on the circle and let the radius vector $OP = I$ be the independent variable in terms of which the equation of the circle is to be derived. Since AP is perpendicular to PK, their scalar or "dot" product is equal to zero. In other words,

$$PA \cdot PK = 0. \qquad (1)$$

Turn PB into the position PB', to coincide with the direction PK. Then we may also write that

$$PA \cdot PB' = 0. \tag{2}$$

But

$$PA = F_1 - I \tag{3}$$

$$PB = F_2 - I \tag{4}$$

$$PB' = (F_2 - I)\epsilon^{-j\delta}. \tag{5}$$

Thus, the equation of the circle is

$$(F_1 - I) \cdot (F_2 - I)\epsilon^{-j\delta} = 0. \tag{6}$$

In this expression, both vectors may be turned by an arbitrary angle θ, so that the equation of the circle may also be written in a more general form as

$$(F_1 - I)\epsilon^{j\theta} \cdot (F_2 - I)\epsilon^{j(\theta - \delta)} = 0. \tag{7}$$

If the electro-magnetic equations of an electric circuit or machine can be reduced to the form (6) or (7), then a circular locus exists, and its parameters, F_1, F_2, δ, become also known. If the electro-magnetic equations cannot be reduced to eq. (6) or (7), no circular locus exists.

Example 1.—Consider the above-mentioned case of a constant reactance x in series with a variable resistance r, connected across a source of steady a. c. voltage E. The fundamental equation is

$$E = rI + jxI \tag{8}$$

or

$$E - j\,xI = rI. \tag{9}$$

The locus equation must contain I only, but not the variable r. To eliminate r, we take a dot product of both sides of Eq. (9) with the vector $-jI$. Since this vector lags behind rI by 90°, the dot product of the two is equal to zero. Therefore

$$-jI \cdot (E - jxI) = 0. \tag{10}$$

Either factor on the left-hand side may be multiplied or divided by a scalar quantity without impairing the validity of the equation. When, however, one of the factors is divided by a complex quantity of argument (angle) α, the other factor must also be divided either by the same complex quantity, or by another of the same argument α. We shall divide the first factor by j and the second by jx, the argument being 90° in both cases. The result is

$$(0 - I) \cdot [(E/jx) - I] = 0. \tag{11}$$

Comparing this expression with Eq. (6), we see that the locus of I is a circle for which $F_1 = 0$; $F_2 = E/jx$, and $\delta = 0$. This means that point 0 co-

incides with A, the diameter AK is equal to E/x and lags behind the vector E by 90°. This agrees with the well known solution of this problem derived from other considerations.

Example 2.—In a series-connected, conductively-compensated, single-phase a. c. motor (Fig. 2) the applied voltage is used up in three parts:

(*a*) In balancing the counter-e.m.f. between the brushes, which e.m.f. is proportional to and in phase with the main flux ϕ. This e.m.f. may be expressed as $av\phi$, where v is the speed of rotation of the armature and a is a constant depending upon the number of turns, etc.

(*b*) In overcoming the reactance of the field winding due to the main flux. This part of the applied voltage may be written as $jb\phi$, where b is a constant of the winding and the factor j shows that the required voltage leads the flux in phase by 90°.

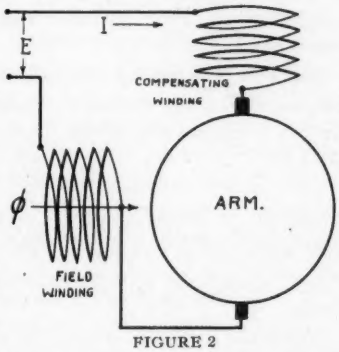

FIGURE 2

(*c*) In overcoming the total leakage impedance Z of the three windings.

Hence, we may write

$$E = av\phi + jb\phi + ZI. \tag{12}$$

The relationship between the flux ϕ and the current I, which excites it, is of the form

$$\phi = CI. \tag{13}$$

Neglecting the core loss, and in a magnetic circuit without saturation, C is a scalar real quantity. With saturation it is a variable real quantity, a function of I. Taking the core loss into account, C is a complex quantity, because the flux then lags in time phase behind the exciting current I. Eliminating ϕ from Eqs. (12) and (13), and separating the term with v, we get

$$E - I(jbC + Z) = avCI. \tag{14}$$

It is desired to obtain an expression for the locus of I without the variable velocity v. Take a dot product of both sides of the equation with the vector $-jCI$ perpendicular to CI. The result is

$$-jCI \cdot [E - IH] = 0 \tag{15}$$

where

$$H = jbC + Z = he^{j(0.5\pi - \eta)}. \tag{16}$$

Let C be a constant complex quantity of the form

$$C = ce^{-j\gamma}. \tag{17}$$

Eq. (15) may be rewritten in the form

$$(0 - I)c\epsilon^{j(0.5\pi-\gamma)} \cdot [(E/H) - I]h\epsilon^{j(0.5\pi-\eta)} = 0. \qquad (18)$$

The scalar quantities c and h may be dropped without impairing the validity of the equation. Comparing Eq. (18) to Eq. (7), we see that the locus is a circle for which $F_1 = 0$, $F_2 = E/H$, and $\delta = \eta - \gamma$. This conclusion is true only provided that both c and γ are constant and independent of I.

Example 3.—For the compensated repulsion motor (*loc. cit.*) the fundamental equations, similar to those established in the preceding example, lead to a relationship of the form.

$$E - IM = INv + IPv^2 \qquad (19)$$

where M, N, P are known constant complex quantities and v is the variable speed of the machine. This equation cannot be reduced to the form (6) or (7), and consequently the current locus for such a machine is not a circle.

From the method used in the elimination of r in Example 1, and of v in Example 2, it will be seen that the most general form of equation which gives a circular locus is

$$E - IM = (IN + P)f(v) \qquad (20)$$

where $f(v)$ may be any scalar function of v. Multiplying both sides by $\cdot j(IN + P)$, the variable v is eliminated, and the resulting equation can be reduced to the form (6) or (7). *Eq. (20) is the desired criterion for a circular locus.* For example, Eq. (19) contains two separate functions of v and is therefore not reducible to the form (20).

Note 1.—To eliminate v from Eq. (19), introduce an auxiliary complex variable D, such that

$$E - IM = ID. \qquad (21)$$

Then, after division by I, Eq. (19) becomes

$$D = Nv + Pv^2. \qquad (22)$$

Multiplying by $\cdot jN$ and by $\cdot jP$, respectively, gives

$$D \cdot jN = v^2P \cdot jN \qquad (23)$$

$$D \cdot jP = vN \cdot jP. \qquad (24)$$

Substituting the value of v from Eq. (24) in Eq. (23), we get

$$(D \cdot jN)(N \cdot jP)^2 = (D \cdot jP)^2(P \cdot jN). \qquad (25)$$

This is the equation of the non-circular locus for D. Substituting scalar

quantities and cosines of the angles, this equation may be readily converted into one in polar coördinates. Knowing D, the corresponding values of I may be obtained from Eq. (21).

Note 2.—A similar general study of circular and non-circular loci, although from a different point of view and by a different method, has been made by Dr. Hermann Pflieger-Haertel. See *Arch. f. Elekt.*, 1923, Vol. 12, p. 486; 1924, Vol. 13, p. 396; 1925, Vol. 14, p. 425.

Summary.—A general equation of the circle is deduced, using the Vector Analysis notation. Electromagnetic vectorial equations of a stationary circuit and of a revolving electrical machine, at a voltage E, are written, and it is shown how to eliminate one of the variables, v, and to reduce the resulting equation for the current I to that of a circle. The general criterion for a circle is shown to be of the form $E - IM = (IN + P)f(v)$, where M, N, P are complex quantities. The compensated repulsion motor is shown to be one of the types of machines for which the locus is not a circle.

[1] Bedell and Crehore, *Alternating Currents*, part on graphical treatment.

[2] E. Arnold, *Wechselstromtechnik*, **5**, part 2, p. 443.

[3] *Ibid.*, **5**, part 1, p. 80.

[4] V. Karapetoff, "The Use of the Scalar Product of Vectors in Locus Diagrams of Electrical Machinery," *Am. Inst. of Elec. Engrs., Journal*, **42**, 1181 (1923). In the forthcoming third edition of Vol. II of the author's "Experimental Electrical Engineering" this method is used in deriving some circle diagrams; see §953 and the references given therein.

POSTULATES FOR REVERSIBLE ORDER ON A CLOSED LINE (SEPARATION OF POINT-PAIRS)

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The type of order called "Reversible Order on a Closed Line"— that is, the type of order which characterizes the "straight line" of projective geometry—presents an exceptionally rich field for the cultivation of examples of pure deductive logic. A long paper which I presented to the American Mathematical Society on April 11, 1925, and which will appear later in one of the mathematical journals, contains the proofs of over two hundred fifty theorems, exhibiting all varieties of elementary logical processes. The paper starts with a list of seventeen basic properties, or postulates, which, though immediately suggested by geometric intuition, are strictly "abstract" in form, and the theorems referred to present an exhaustive account of the logical inter-relations which exist among these

seventeen postulates. A study of these theorems for the purpose of eliminating all redundancies from the basic list leads to several shorter lists of "independent" postulates, any one of which might serve as the basis for a deductive development of the theory of order on a closed line.

The simplest of these sets of independent postulates is believed to be new, and since this set may be of service to working mathematicians apart from the elaborate logical discussion in the longer paper, it is here presented in separate form.

We are dealing with a class of elements, A, B, C, ..., which may be thought of as "points," and a tetradic relation, R(ABCD), or simply ABCD, which may be thought of as the relation of "order." To avoid trivialities, we assume once for all that the class contains at least four elements and that the letters A, B, C, D, in any tetrad ABCD, are distinct.

Concerning all the possible tetrads which automatically exist among the elements of the class, we state first a preliminary postulate, which serves merely to exclude obviously trivial cases:

POSTULATE 0. *At least one tetrad is true, say XYZW.*

The general properties of order on a closed line are then expressed in the following three postulates:

POSTULATE G. *If ABCD is true, then BCDA is true.* This is the fundamental property of "cyclicity."

POSTULATE H. *If ABCD is true, then ABDC is false.* This property may be called "homogeneity."

POSTULATE 10. *If ABCD is true, and X is any fifth element, then at least one of the relations, AXCD and ABCX, is true.* This property may be called "connexity," since it excludes the case of isolated or disconnected elements.

The following postulate serves to distinguish the reversible from the irreversible type of order on a closed line:

POSTULATE R′. *At least one true tetrad is reversible.* That is, if any tetrad is true, then at least one true tetrad, say XYZW, is such that WZYX is also true.

From the first four of these postulates, the following theorem, which has usually been assumed as a fundamental property, can be deduced.

THEOREM F. If A, B, C, D are any distinct elements, then at least one of the twenty-four permutations, ABCD, ABDC, ..., DCBA, will form a true tetrad ("Foursomeness").

It will be observed that Postulate 0 is a much weaker statement than Theorem F.

From the five postulates, 0, G, H, 10, and R′, it follows that *every* tetrad is reversible; that is, we have

THEOREM R. If ABCD is true, then DCBA is true.

The proofs of these two theorems are somewhat analogous to the proof of Theorem 203 in my paper on "A New Set of Postulates for Betweenness," *Transactions of the American Mathematical Society*, Volume 26, April, 1924, page 279.

Two other theorems which may be mentioned are the following:

THEOREM 11. If ABXC and ABCY, then ABXY.

THEOREM 14. If ABCX and ABCY are true, then at least one of the tetrads ABXY or ABYX will be true.

THEOREM 17. If ABCX and ABCY are true, then at least one of the tetrads ABXY or ACYX will be true.

Each of these theorems is readily verified from a figure. The main interest in this and similar theorems from the present point of view is not that they are "true," but that they are deducible in the abstract sense from the postulates stated without reference to a figure or to any other concrete interpretation of the symbols.

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*THE HYPOTHESIS OF INHIBITION BY DRAINAGE**

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Statement.—The drainage hypothesis assumes that there is in the nervous system and in each of its elements a definite amount of available neural energy which can be concentrated into specific neural paths and consequently drained from others. The inhibition of a process occurs when the energy that previously conditioned it is drained off into other channels. In the history of the theory of inhibition this hypothesis goes back to Herzen and Schiff[1] and to Setschenow.[2] It was developed at some length by Alexander James[3] and William James[4] but reached its completest statement and scientific justification in the presentation of William McDougall[5] to whom it is commonly credited. The hypothesis seems to fit a large number of physiological and psychological phenomena. McDougall believed it was confirmed by test experiments in the fields of contrast and biretinal rivalry.

Theoretical Difficulties.—Since the hypothesis was first elaborated there

has been a notable change in the scientific tradition concerning the nature of neural action. It has become increasingly probable that each element of the nervous system reacts maximally or not at all to quantitative and qualitative changes in its external vital conditions, the energy of its reaction bearing no fixed relationship either to the energy of the stimulation to which it responds or to the energy of the processes that it sets in motion. This tradition is diametrically opposed to a hydrodynamic interpretation of neural action. It appears quite improbable that neural energy flows in any hydrodynamic sense, that it can be diverted into specific paths, damned up, or drained.

In addition to the evidence that each neural element reacts to stimulation according to the all-or-nothing law it is now probable that neural excitation is propagated without decrement throughout each axone and its system of collaterals. If this is true there seems to be no inner mechanism for apportioning the intensity of propagation between the collaterals and the main axone and no opportunity for the receptor pole of a neurone to drain into a single collateral the energy of the effector pole of its stimulator.

Experimental Test.—The final test of the hypothesis, however, is not its congruence or lack of congruence with more or less probable scientific tradition but its correspondence with experimentally controlled facts. McDougall's tests involving visual contrast and biretinal rivalry are in a field where control is difficult and interpretation hazardous. If the drainage theory were otherwise demonstrated they might be regarded as interesting examples. They lacked the criteria of a crucial test. Shepard and Fogelsonger's[6] experiments in association demonstrated that drainage is not the exclusive mechanism of inhibition, since there were no related pathways in which drainage could occur.

The main question at issue in a crucial test of the drainage hypothesis may be stated as follows: Is it, or is it not, a fact as is assumed in the drainage theory that the inhibition of a reflex regularly occurs as a correlate of increased spread of excitation in superior neural systems? That is to say, is a reflex inhibited by voluntary reaction to the stimulus that evokes it?

This formulation of the question indicates use of the intact human nervous systems where voluntary reactions and reflexes are both available. Experiments were consequently made on two reflex systems, the knee-jerk and the wink reflex. The knee-jerk was elicited by a blow on the patellar tendon,[7] the wink reflex by a sound stimulus.[8] The techniques in both cases have been adequately developed and involve no significant instrumental errors. In the knee-jerk the experimental spread of cortical excitation was provided for by the voluntary release of a telegraphic key in series with the recording instrument. The reflex was neither eliminated nor measureably reduced. Fearing that the hypothetical drainage might have been reduced by the disparateness of the neural systems which were

involved, I modified the wink reflex experiment so as to utilize not only a common stimulus, but a final common path terminating in the orbicularis. In this case the spread of excitation was provided for by voluntary closure of the lid. Notwithstanding these precautions the lid reflex was neither eliminated nor diminished in extent by the spread of excitation and subsequent voluntary closure. The agreement of the results in both reflex systems gives them a considerable degree of reliability.

Conclusion.—The theoretical difficulties and the experimental tests combine to render the drainage theory of inhibition improbable in the human neural system. Unless new evidence is discovered, inhibition by drainage ceases to be available as a principle of interpretation of human behavior.

* A more detailed statement of the drainage theory and its difficulties will appear in an early number of the *Psychological Review.*

[1] Herzen and Schiff, *Expériences sur les centres moderateurs de l'action reflexe, Turin,* 1864.

[2] Setschenow, *Physiologische Studien über die Hemmungsmechanismen ect., Berlin,* 1863.

[3] James, Alexander, *Brain,* **4,** 287 (1881–2).

[4] James, Wm., *Principles of Psychology, New York,* 1896.

[5] McDougall, Wm., *Brain,* **26,** 153 (1903). *Physiological Psychology, London,* 1908.

[6] Shepard and Fogelsonger, *Psych. Rev.,* **20,** 290 (1913).

[7] Dodge, *Zeit. allg. Physiol.,* **12,** 1 (1910). Dodge and Benedict, *Psychological Effects of Alcohol, Washington,* 1915.

[8] Dodge, *Amer. J. Psych.,* **24,** 1 (1913).

A TEST OF THE ELECTROMAGNETIC THEORY OF THE HYDROGEN VORTICES SURROUNDING SUN-SPOTS

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The extensive fields of force shown by the spectroheliograph in the hydrogen atmosphere surrounding sun-spots have been explained in two different ways: (1) as true hydrodynamical vortices, resembling great tornadoes, and (2) as electromagnetic phenomena, in which charged particles moving in the solar atmosphere are constrained by the magnetic fields in the spots to follow their lines of force. The principles involved in the electromagnetic theory have been applied to the explanation of the terrestrial aurora by Störmer, who has also developed this theory for the case of sun-spots.[1]

Störmer starts with the assumption that the magnetic field in a sun-spot is due to electrically charged gas molecules, moving at low levels along spiral paths around the center of the spot. In the numerical example given in this paper the thickness of this plane whirl is regarded as only a small fraction of its diameter, which is assumed to correspond with the diameter of the high-level whirl revealed by the hydrogen flocculi (generally ranging from about five to twenty times that of the spot). Störmer concludes that the projections of the lines of force of the sun-spot field (corresponding in his theory to the whirls in the hydrogen flocculi) are logarithmic spirals cutting the current lines of the underlying (invisible) sun-spot vortex everywhere at right angles. The direction of the apparent whirl shown by the hydrogen flocculi (curvature of the projected lines of force), clockwise or counter-clockwise, should then depend upon the sign, motion and direction of the invisible low level electric current surrounding the spot.

On the assumption that this current represents the flow of negatively[2] charged particles, five cases are possible:

(1) If the flow is circular around an axis corresponding to a solar diameter through the center of the spot, the lines of force will be plane curves situated in planes passing through this axis. Their projections on the solar surface will be straight lines radiating from the center of the spot, and the hydrogen flocculi should, therefore, appear as straight radial lines, if the spot is near the center of the sun.

(2) If the flow is spirally outward in the clockwise direction, the projected lines of force will show a clockwise curvature and the north pole of the spot will be upward; if the flow is spirally outward and counter-clockwise, the curvature of the projected lines of force will be counter-clockwise and the south pole of the spot will be upward.

(3) If the flow is spirally inward and counter-clockwise, the curvature of the projected lines of force will be clockwise and the south pole of the spot upward; if spirally inward and clockwise, the curvature of the projected lines of force will be counter-clockwise and the north pole of the spot upward.

Clockwise curvature, as used above, are such that if a point moves inward along the lines of force, the rotational component of its motion is in a clockwise direction. If the rotational component of the inward motion is counter-clockwise, the curvature is counter-clockwise.

Our studies of the Zeeman effect in sun-spots show that the angle between the lines of force and the solar surface, which is about 90° at the center of the spot, decreases gradually to about 0° near the outer edge of the penumbra, beyond which the field becomes too weak to be detected by the methods hitherto employed.[3] The observed field, therefore, does not correspond with the field of much greater diameter postulated by Störmer in his numerical example, but indicates that if a low level current is its cause,

this must be confined to the region within the penumbra and an extremely narrow zone surrounding it. Moreover, our photographs of the hydrogen vortices almost invariably show a practically radial structure near the spot, even when the curvature is very marked at greater distances. In the shallow layer of the chromosphere, to which the spiral structure studied in this investigation is confined, the projections of the lines of force due to a circular current would be practically straight radial lines. Even if the curved lines in vertical planes extending to higher levels were in question, as they might be in prominences, it is difficult to see how their projections could simulate the observed vortex structure or avoid betrayal by their changes in form as they are carried across the disk by the solar rotation.

If, however, in spite of the limited extent of the magnetic field and the radial structure of the hydrogen flocculi near spots, the electromagnetic theory can be modified so as to call for the observed vortices, the direction of curvature of the lines of force, clockwise or counter-clockwise, must nevertheless conform with the polarity of the spots in question if two assumptions are permissible. These assumptions are that the sign of the charge of the particles producing the field and the direction of any radial component of motion, inward or outward, are invariable in all single spots and in the preceding spots of bipolar groups. The long series of observations from which we have deduced the law of sun-spot polarity points to the invariability of the sign of the charge producing these sun-spot fields and the reversal of the direction (clockwise or counter-clockwise) of these low level whirls at every sun-spot minimum.[4] The question whether the motion in these whirls (at the level where the magnetic field is produced) is approximately circular or has a radial component is not yet settled, but there is no reason to suppose that any such radial component changes from inward to outward or vice versa. Assuming its direction, toward or from the spot, to be invariable, we may proceed to apply the polarity criterion. This is easily done, as the polarities of all spots are recorded at Mount Wilson, while the hydrogen vortices are also photographed there daily.

In spite of the thousands of spectroheliograms available, the number of hydrogen vortices suitable for the test is comparatively small. The great majority of spot-groups must be rejected, because of the disturbing influence of their numerous components, frequently of opposite polarities; the effects of turbulence in the solar atmosphere, especially near active groups; or the variation in the curvature of the stream lines with altitude, which may cause only the radial structure corresponding to a particular level of maximum Hα absorption to appear.

For the purposes of this test, I have selected at random 51 of the best hydrogen vortices on the Mount Wilson negatives, made on various dates scattered over the period 1908–1924. This permits the inclusion of vortices associated with spots belonging to three successive $11^1/_2$-year

cycles, covering two complete reversals of magnetic polarity.[3] Both unipolar spots and the preceding members of bipolar groups are represented, but all vortices are excluded in which there is uncertainty regarding the direction of whirl. The motion of the gases at the Hα level is inward toward the center of the vortex, and the terms clockwise and counter-clockwise correspond with the conventions used in the case of terrestrial cyclones.

UNIPOLAR SPOTS

	NORTHERN HEMISPHERE			SOUTHERN HEMISPHERE		
CYCLE	NO. OF SPOTS	POLARITY	WHIRL	NO. OF SPOTS	POLARITY	WHIRL
First	3	S	Counter-cl.	4	N	Clockwise
Second	8	N	Counter-cl.	6	S	Clockwise
	3	N	Clockwise			
Third	4	S	Counter-cl.			

BIPOLAR SPOTS

	NORTHERN HEMISPHERE			SOUTHERN HEMISPHERE		
CYCLE	NO. OF SPOTS	POLARITY OF PREC. SPOT	WHIRL PREC. SPOT	NO. OF SPOTS	POLARITY OF PREC. SPOT	WHIRL PREC. SPOT
Second	8	N	Counter-cl.	6	S	Clockwise
	2	N	Clockwise	3	S	Counter-cl.
Third	3	S	Counter-cl.			
	1	S	Clockwise			

Summarizing the polarities and directions of whirl without regard to date, spot-type, hemisphere or latitude, we find

NO. OF SPOTS	POLARITY	DIRECTION OF WHIRL
9	N	Clockwise
16	N	Counter-clockwise
13	S	Clockwise
13	S	Counter-clockwise

Evidently no relationship between polarity and direction of whirl is shown by these spots, taken at random from three $11^1/_2$-year cycles. The first table also indicates that after the reversal of the magnetic polarity of the spots at two successive minima[4] there was no corresponding reversal in the direction of whirl in the associated hydrogen vortices. The results therefore do not seem to support the electromagnetic theory.

Grouping for each hemisphere the whirls associated with unipolar spots and with the preceding members of bipolar spots, we find the following directions of whirl:

	NORTHERN HEMISPHERE	SOUTHERN HEMISPHERE
Unipolar Spots	15 counter-clockwise	10 clockwise
	3 clockwise	
Bipolar Spots	11 counter-clockwise	6 clockwise
	3 clockwise	3 counter-clockwise

Thus 81 per cent of the northern vortices and 84 per cent of the southern

vortices, irrespective of the $11^1/_2$-year cycle in which they occur, agree in direction of whirl with terrestrial cyclones. This agreement, taken in conjunction with the test of the electromagnetic hypothesis and the motions in the solar atmosphere above spots observed by Evershed and St. John, suggests that the hydrogen vortices are hydrodynamical phenomena, and that their direction of whirl is generally determined, not by the direction of whirl of the sun-spot vortices below them, but by the eastward and westward deflection, resulting from the solar rotation of currents flowing northward and southward in the solar atmosphere toward centers of attraction above sun-spots.

The exceptions may be due to the following causes, acting singly or in conjunction:

(1) The small relative drift resulting from the slow change with latitude of the linear velocity of the atmosphere caused by the sun's rotation.

(2) The turbulence of the solar atmosphere, especially near active spots, which may prevent the formation of an induced vortex, mask its structure, or determine its directions of whirl.

(3) The fact that about three-quarters of all sun-spots, at the time of their formation, are either unipolar or bipolar groups in which the preceding member is larger than the following member. In such cases, according to the working hypothesis now being tested, the direction of whirl of the hydrogen vortex induced over unipolar spots or over the preceding member of bipolar groups should, in general, follow the law of terrestrial storms. In cases where the following spot is larger than the preceding spot at the time of formation, the direction of the hydrogen whirl above the following spot should, in general, conform with the terrestrial law, while the preceding whirl should be of opposite sign if the influence of the following whirl outweighs that of the rotational drift. In bipolar groups where both spots are equal at the time of formation, each spot has an equal chance of starting a whirl following the terrestrial law, and local conditions may determine the outcome.

By means of a series of vortex experiments, continuing those previously described in these PROCEEDINGS,[5] I have been able to imitate many of the characteristic phenomena of the hydrogen vortices surrounding unipolar and bipolar spots. At a distance from the spots these vortices often closely resemble logarithmic spirals, which also characterize terrestrial cyclones. The tendency of the hydrogen vortices to show a rapid increase in the radial component of motion (suggesting marked acceleration) in approaching the spots is, however, so general that it deserves special consideration. Full details of this investigation will be given in the *Astrophysical Journal*.

I am indebted to Messrs. Nicholson and Ellerman and to Mrs. Humason for valuable assistance in connection with this paper.

[1] Störmer, "Researches on Solar Vortices," *Contributions from the Mount Wilson Observatory, No. 109; Astrophysical Journal*, **43**, 347 (1916). The electromagnetic theory of solar vortices was developed by Professor Störmer merely as a working hypothesis and not as an expression of his opinion regarding the nature of these phenomena. The legend of figure 9 in Professor Störmer's article is incorrect: the words "clockwise" and "counter-clockwise" should be transposed.

[2] This sign is not known. If positive instead of negative, the polarity of the spots in the cases given will be reversed. Near the end of his paper Störmer concludes that galvanic currents are more probable than convection currents of charged particles.

[3] Hale, Ellerman, Nicholson and Joy, "The Magnetic Polarity of Sun-spots," *Contributions from the Mount Wilson Observatory, No. 165; Astrophysical Journal*, **49**, 165 (1919).

[4] Hale, "The Law of Sun-spot Polarity," these PROCEEDINGS, **10**, 53 (1924).

[5] Hale and Luckey, "Some Vortex Experiments Bearing on the Nature of Sun-spots and Flocculi," these PROCEEDINGS, **1**, 385 (1915).

RECENT EVOLUTION IN MILK SECRETION OF GUERNSEY CATTLE

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Under the title of evolution much has been written with regard to the development of the morphological characters in the plant and animal. While such studies will undoubtedly remain the foundation on which the theory of evolution must rest for many years to come, it is of interest to view the progress of evolution in some of the characters which surround us and are familiar in everyday life. Furthermore it is important to consider the evolution of those parts which have to do with the everyday physiology of the animal rather than its more nearly structural form.

In the milk production and butter-fat concentration of milk we have an opportunity to make such a study of the progressive effect of the factors contributing to evolution of a given class of animals. These factors may be those which are innate and due to a selection of a better heredity for the animals making successive generations, or they may be due to those sociological causes found in the advance of our scientific knowledge on the effect of environment on the expression of the animal's innate capacities. Ordinarily it is difficult, if not impossible, to separate these two variables in any study. In dairy cattle it is doubly difficult in view of the rapid progress which has taken place recently in our knowledge of their proper feeding and management. For the purpose of this study it is proposed to show the changes which have taken place in Guernsey Advanced Registry

cattle from the time of the inception of the Advanced Registry to the present day.

Milk is, of course, a secretion of one of the complex body glands. The concentration of the fat in the milk represents the variation in the percentage of a product which may be found in such a secretion. Figure 1 shows the progressive changes in the milk production, butter-fat percentage and butter-fat of Guernsey Advanced Registry animals from the time of the establishment of the Advanced Registry to the present day. For this purpose the different periods in the evolution of these cows are indicated by their Herd Book numbers. The first averages represent the results for those cows having 1 to 5000 as their Herd Book numbers. The second group, the averages of those cows from 5000 to 10,000 of Herd Book numbers. In this way a picture of the changes which have taken place in the productivity of Guernsey cattle are indicated against the changes which have taken place in time.

The two upper charts in figure 1 are surprising. They show no increase in either milk yield or butter-fat percentage during the different periods of registration of Guernsey cattle, whether these productions be taken as the highest, the average or the lowest production for the given period. The question, however, is not settled by these charts as from the third chart it will be noted that a marked age change has taken place in the animals entered for Advanced Registry during the different periods. The solid line represents the average age (as given on the right-hand side of the figure) of the cows entering the Advanced Registry for any given period. It will be noted that during the early days of the Advanced Registry only older cows were entered. There has been since this time a progressive change towards testing cattle early in life, making a constant decline in the age of the cows which have been tested for the Advanced Registry within the different periods of time. The dotted line shows the frequency distributions for the number of animals shown at the left for each of the given periods of time within which the Advanced Registry data were divided.

It is known that age materially affects the production of milk in dairy cattle. This effect of age is represented by a marked but ever declining increase in production as the cow advances in years until a maximum is reached, when at about 7 to 9 years the cow's production commences to decline at an ever increasing rate. To a less degree the butter-fat is also affected by age, age resulting in a slight decline in the butter-fat percentage of Guernsey cattle. From these facts it is clear that this decline in the age of production of cows entering Advanced Registry is an important item in considering the interpretation to be given to the data in figure 1. It is, in fact, surprising rather than otherwise that the average production of Guernsey cattle should have remained the same during the different periods of registration as shown in Chart 1 rather than having declined in

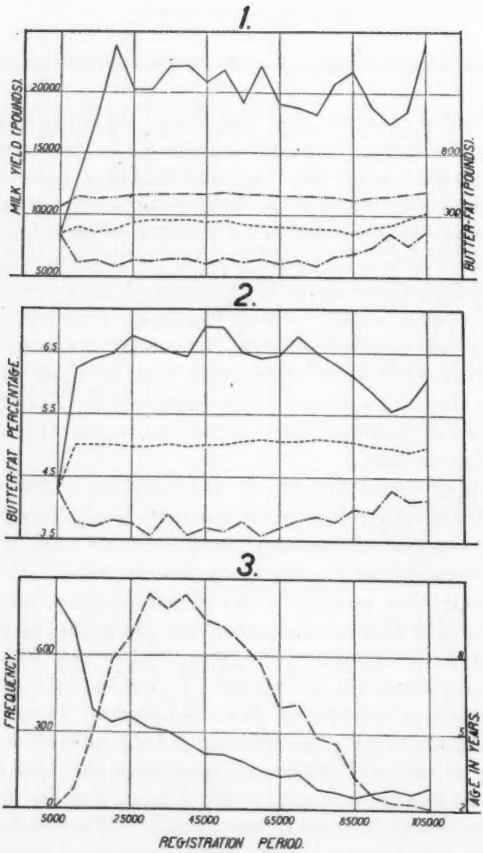

FIGURE 1

Progressive changes in the average age-corrected milk yields and butter-fat percentages of animals making the Guernsey Advanced Registry.

Chart (1) gives the data for milk and butter-fat yield. The cows making the Advanced Registry are divided into periods according to their Herd Book numbers. The age-corrected record of highest producing cow during each period is shown as the solid line; the average uncorrected milk yield as the dotted line; the lowest age-corrected milk yield as the dot and dash line; the average uncorrected butter-fat yield as the dot, dot, dash line.

Chart (2) shows similar data for the butter-fat percentage of these cows. The solid line represents the highest age-corrected butter-fat percentage of these cows for each period; the dotted line the average uncorrected butter-fat percentages for each period; the dot and dash line the lowest age-corrected record for each period.

Chart (3) shows the average ages of the cows within each group of cows as the solid line. The dashed line shows the number of cows in each group. It is clear from these charts that the age of the cows making the Advanced Registry at these different periods has changed markedly. Thus the dotted lines showing the average milk yields and butter-fat percentages of the cows for the different periods does not present a true picture of the events.

production, for, as indicated, younger cows, in general, milk less than do older cows.

This difficulty may be overcome by placing the records on an age corrected basis, or in other words, correcting the production of the cow to what it would have been at mature form, 8 years and 3 months. Figure 2

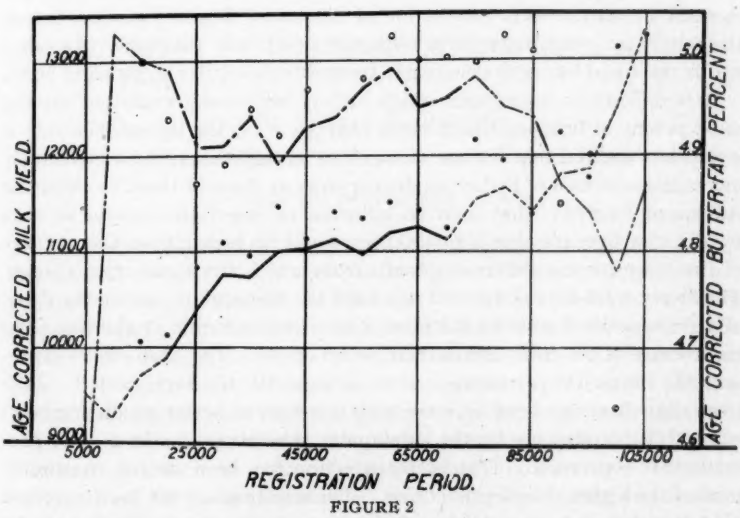

FIGURE 2

Progressive changes in Guernsey Advanced Registry Cows from the time the advanced registry was established to the present day

The dot and solid line represents the age-corrected milk yields for each period in the breed's history. The dot, dot, dash and the dot, dash line represents the age-corrected butter-fat percentages during each period. The heavy lines throughout represent the data to be given most weight. The solid circles give the average age-corrected milk yields of the recorded dams of the cows registered at the given period. The open circles the average age-corrected butter-fat percentages of the recorded dams of the cows registered at the given period.

From figures 1 and 2 the following conclusions may be drawn. (*a*) Within the significant period, 20,000 to 70,000 of Herd Book numbers, there has been no significant change in the milk yields or butter-fat percentages of the highest and lowest cows. (*b*) During this time there has been a continuous decrease in the age of the cows tested. (*c*) There has been a significant increase in the average age-corrected milk yields of these cows during the period. (*d*) These cows appear to come from the better type of dams. (*e*) No significant change has taken place in the butter-fat percentage.

shows the result of making these corrections. It will be noted that on a mature form basis there has been a constant increase in the milk production of Guernsey cattle. No such increase is to be noted in the butter-fat percentage. In fact, in general, the butter-fat percentage is practically the

same throughout the whole period. Within these charts the milk yields are shown as the solid and dotted line, the butter-fat percentages as the dot and dash line, the heavy lines represent the data which are significant, the light lines those of doubtful value due to lack of numbers of individuals. Confining our attention to the significant data represented by the heavy lines it will be noted that in milk yield there has been a constant increase in mature form production of Advanced Registry cows, whereas the butter-fat percentages have remained practically the same. In other words the trend has been constantly toward higher milking, younger cows.

It is difficult to distinguish which factors influencing evolution are the most potent in bringing about these changes. On the inheritance side it would be expected that if these changes are brought about by a constantly increasing selection of higher producing cows as dams of those to enter the Advanced Registry, that such an influence of inheritance would be seen by the fact that the dam's production would be higher than the average of the cows for the different periods from which the dams were chosen. The heavy solid dots of figure 2 represent the amounts by which the dams of a given series of cows were higher or lower in their milk production than the average of the cows within their period of life. The open circles represent the butter-fat percentages of these dams for the same period. It is clear that there has been an increasing selection of better producing cows to furnish the offspring for the subsequent generation insofar as milk production is concerned. This better selection has been almost constantly toward the higher side of production. The selection has not been as severe for butter-fat percentage, although, in general, the selection for butter-fat percentage has been somewhat above the average for all cows.

From these facts it is seen that the selection of better cattle as dams to those making the Advanced Registry has probably played a part in increasing the milk production as shown by these Advanced Registry cattle. On the other hand, it is doubtful if this better inheritance is the only factor in bringing about this increase. It seems more probable that the increased knowledge of feeding and the care of dairy animals has also played a large part in the increase of production. In view of the negative relation which exists between amounts of milk yield and the butter-fat percentage of the milk we would expect a decline in the butter-fat percentage. However, the butter-fat percentage has changed but little. The influence of better heredity and the increased knowledge of feeding and care of dairy cattle has been able to nearly maintain the butter-fat percentage constant while the milk yield was increased in amount.

Here, within present day remembrance, is presented a picture of progressive evolution in the secreting power of one of the most important body glands. For that portion of the gland's activity subjected to agencies which are believed to bring about progressive evolution, there is a marked

increase in the average ability of the cows within the Guernsey breed, namely, an increase in the milk producing capacity of these animals. For that portion of the secretion where these agencies of evolution have been less active the original level of activity has been maintained.

ELIMINATION OF CHROMOSOMES DUE TO A MUTANT (MINUTE-N) IN DROSOPHILA MELANOGASTER

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The mutant character Minute-n in Drosophila melanogaster belongs to the general category of Minutes (see Bridges and Morgan, *Carnegie Publication*, No. 327, page 207), in that (1) the bristles throughout the fly, especially those on the thorax and head, are shorter and slenderer than those of the normal wild-type fly. Likewise the character is (2) a dominant. The gene is (3) lethal when homozygous. And (4) the development to emergence from the pupa takes two or three days longer than that of the wild-type sibs. Besides the above key characteristics of Minutes, Minute-n flies show to a variable extent other characteristics common to Minutes, as, for example, smaller size, abnormal bands on abdomen, reduced aristae, irregular venation, paler body-color, frequent sterility of the females and rather low productivity of some of the females that are fertile, etc. All these characters are inherited as a unit group, and are due either to a single point-mutation, or, more probably, to a section-mutation or "deficiency" (Bridges, *Genetics*, **2**, 445–465). Minute-l, situated near the right end of chromosome II, is known to be a deficiency for the loci of arc and plexus that are about a unit apart, and for the section between them (*Carnegie Year Book*, No. 23, p. 233). The locus of Minute-n is in the X-chromosome, about six units to the right of forked, or at approximately 62. Since the locus of Minute-n is in the X-chromosome, the character occurs only in females. In the male it is lethal. This agrees with the Notch-deficiencies. The other two Minutes in the X, viz., Minute-k and Minute-o, are likewise dominant in the female and lethal in the male. The termination -n of the name Minute-n designates that this was the fourteenth mutant of the Minute type (found Jan. 11, 1924) to be studied extensively enough to be given a distinctive name. If Minute-n is a deficiency, it nevertheless does not produce changes in crossing-over extensive enough to interfere with its use in linkage studies. Since the locus of Minute-n lies to the right of forked it has provided a means of deter-

mining experimentally for this region the course of the curves that show the relation between map-distance (total crossing-over per hundred reductions) and recombination (total recombination of the linked characters—less than crossing-over because double crossing-over does not result in recombination of the characters). Formerly these curves were extended to the region to the right of forked mainly by extrapolation (*Carnegie Year Book*, No. 23, p. 235).

In crossing Minute-n females to yellow males it was found that occasionally a Minute-n daughter occurred that had a small spot, or even half of the dorsal surface of the thorax, yellow, instead of wild-type in

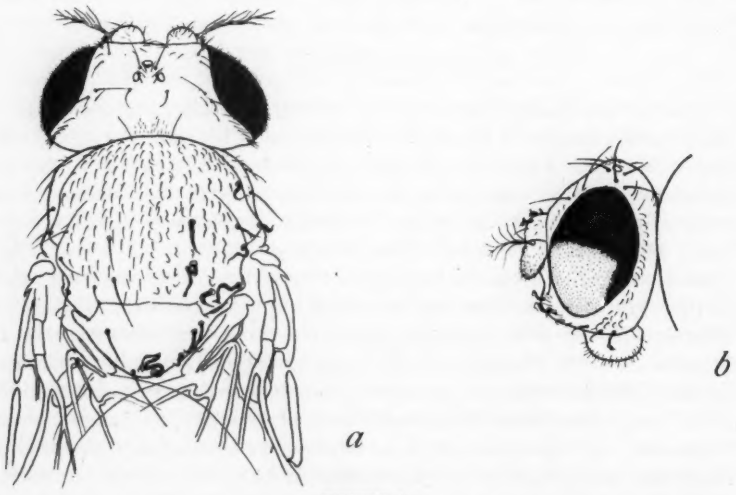

FIGURE 1

"Piebalds." Fig. 1*a* shows a Minute-n female in which the spot (singed but not Minute-n) covers the right dorsal half of the thorax. Fig. 1*b* shows a piebald in which the left eye has a spot of male-type eosin color with the adjacent tissue singed. An independent spot of singed occurs on the antenna.

body color, and with normal sized bristles, instead of Minute bristles. When the males used had forked bristles as well as yellow body color the "piebald" daughters showed both yellow and forked in the not-Minute spots. Other sex-linked body-colors (sable, silvery) and bristle characters (scute, Hairy-wing, singed) were tested and it was found that the spots always showed the sex-linked character or characters that the father carried.

In the studies of gynandromorphs and mosaics in Drosophila (Morgan and Bridges, *Carnegie Publication*, No. 278), many individuals had been found of the type in which most of the tissue was biparental in character-

istics but in which a section of the body showed the sex-linked characters that the father, or the mother, had carried. In those cases the patroclinous tissue had been male, and the other tissue female. The explanation advanced for the bulk of such gynandromorphs, and verified in numerous cases since, is that they started with a set of maternal chromosomes, and a set of paternal chromosomes, each set containing an X-chromosome. The individual was therefore a diploid hybrid female, and would be expected to show the characters corresponding to the dominants of the pairs of characters involved in the cross. At an early cleavage stage (probably at the first or second division of the fertilized egg nucleus) there was "elimination" of one daughter member of the dividing maternal chromosome. One daughter member reached a daughter nucleus normally, but the other was lost in the middle of the spindle between the nuclei. The other X, the paternal X, divided normally and each daughter nucleus received a member. One nucleus would then remain as before and give rise to tissue that would be hybrid and female; the other nucleus would have one X with two sets of autosomes and would give rise to tissue that would be male and patroclinous with respect to all sex-linked characters. In the gynandromorphs the elimination occurred very early, for usually approximately half, or approximately a quarter, of the fly was male.

It seemed that the piebald Minutes were likewise mosaics due to elimination of a maternal X-chromosome. While the earlier type of gynandromorph was as often matroclinous as patroclinous in the male part, in the piebalds the aberrant part seemed always to be patroclinous. In order to test this further, the same characters used before (yellow, sable, forked, singed, etc.), were combined with Minute-n and introduced through the mother. In the piebald offspring of such crosses the spots never showed these characters but always the characters of the father. Apparently therefore the elimination was not according to chance, but the chromosome carrying the Minute-n gene, was frequently eliminated.

The spots of the piebald should be male in constitution if they are due to elimination of one X-chromosome. Male parts of gynandromorphs are in general smaller than corresponding female parts. The spots of the piebald were not smaller than the surrounding female tissue, but most of the spots were too small for a size difference to show readily. The Minute-n fly is itself somewhat smaller than a normal female, and this would balance the smallness due to the maleness of the not-Minute parts.

Several thousand Minute-n females have been examined to find a piebald in which the spot included the comb region of the foreleg. If the spots were male this would be indicated by the presence of a sex-comb. The spots could be detected on the leg through use of yellow body-color and forked bristles in the father. But apparently this region was one in

which the elimination was very rare indeed. Only one individual was found in which a spot involved a foreleg. This individual differed in other characteristics, which indicated that it was a mosaic triploid for all the chromosomes rather than a piebald of the ordinary type. This mosaic will be described in detail in another paper.

In the crosses to singed, a piebald was found in which the spot covered nearly half of the abdomen laterally. The posterior part of this region was colored black as in the male. Only this one specimen has been found in which the spot involved the abdomen.

Frequently part of an eye or even nearly a whole eye was involved. This gave opportunity to test the sex of the spot by using the sex-linked eye-color eosin. The eye color of females of a pure stock of eosin is pink, only slightly yellowish in tone. The eye color of the eosin male is yellow, only slightly pinkish. This difference was known to hold in the female and male parts of gynandromorphs. When Minute-n females were mated to eosin singed males it was found that the spots in the eye had the male eosin color. More than thirty piebalds have been seen in which the spot of eosin was large enough for the eye color to be easily judged, and they all agreed in being of the male type. Figure 1*b* shows a typical piebald in which the eye spot was male eosin in color and the neighboring bristles and hairs were not-Minute but were singed.

The ordinary piebald was supposed to be due to the elimination of the maternal Minute-n X-chromosome without any accompanying elimination of the maternal autosomes. A test was made as to whether the tissue of the spot was diploid or haploid with respect to the autosomes. Males that were eosin and singed and also homozygous for the third-chromosome recessive ebony body-color were mated to Minute-n females. As before, the spots were singed, and those in which the eye color was involved showed the male type of eosin (except one individual in which the spot was female eosin and ebony, and that was interpreted as a haploid mosaic rather than a piebald). The body color of the spots was not ebony, and there was no difference in body-color between the spot and the surrounding Minute-n tissue. Figure 1*a* shows a specimen in which the spot covered nearly half the dorsal surface of the thorax and in which no difference in color or size could be seen. The spots of the piebald were thus shown to be diploid and hybrid with respect to the autosomes but haploid and patroclinous with respect to the X through elimination of the maternal X carrying the Minute-n gene.

In studying the occurrence of the spots, with respect to the region of the body, the size of the spots and their frequency, it was found that very much depended on the check characters used. If the character difference was not sharp, small spots would be overlooked, and the line of demarkation of the larger spots could not be made out exactly. The most useful

mutant characters were singed and forked[5], which affect all the bristles and hairs of the body (see Figs. 1*a* and 1*b*), and the body-color yellow. Yellow could be detected in all parts of the body except the eye, and singed and forked[5] in all parts except the eye and most of the wing. Spots occuring in the eye were checked best by the eye-color white, though eosin was satisfactory and had the added advantage of being an index of the sex of the spot. The bulk of the tests were carried out with two stocks in which favorable characters were combined, viz., yellow white forked[5] and eosin singed vermilion. Additional wing characters that were used to some extent were cut, cross-veinless, and Hairy-wing. Additional eye characters were garnet and echinus.

It was found that the most frequent, or perhaps most easily detected, type of spot was that occurring in the eye. Nearly half of all the spots seen were in the eye, though many of these were very small. Spots covering as few as four ommatidia could be detected. The most frequent type of spot was a sector reaching in from the anterior border of the eye. About five per cent of the eye-spots covered as much as third of the eye (Fig. 1*b*) and in these it was easy to see the type of eosin color. Nearly as frequent as the spots in the eye were those found on the dorsal surface of the thorax. Very small spots on the thorax would usually be missed, though they are known to occur, and occasionally spots involving only a single bristle or a tiny patch of hairs have been seen. The most frequent location for the small spots on the thorax was just dorsal to the wing and anterior to the scutellar furrow. Larger spots usually involved this same region. In about five per cent of the dorsal spots an area from mid-line to wing-level and from tip of scutellum to the humeral patch was involved (Fig. 1*a*). Spots on the rest of the head outside the eye, and those on the ventral part of the thorax, were only about a fifth as frequent as those in the first-mentioned zones. The wings, legs and abdomen seemed to be affected but seldom.

The proportion of Minute-n females that were piebalds varied somewhat from culture to culture but usually was between 10 and 40 per cent. In the set of cultures most carefully examined there were 106 piebald individuals in a total of 322 Minute-n daughters, or 33 per cent. Of the 106 piebalds, 68 had a single spot, 28 had two separate spots, 7 had three and 2 had four spots. In figure 1*b* the tiny spot on the antenna is independent of the large spot that involves the eye and cheek.

There was a very decided difference in proportion of individuals that were piebalds in the first counts of a culture as compared with the later hatches from the same culture. It was not unusual for the first Minutes that hatched to show over fifty per cent piebalds, while it was rare for the final counts to show over five per cent of piebalds. Evidently the elimination was affected by the environment to a marked degree, as shown not only

by the highly preferential regional location of the spots, but also by the change in frequency with change in the external conditions.

The small size of most of the male spots, and the fact that usually only part of an organ coming from a single imaginal disc was affected, showed that the elimination occurred late in larval development, after these discs have completed part of their development. A careful and detailed study of the size and distribution of the spots should give much information concerning the cell lineage in dipterous development.

One of the stocks used extensively in studying the size, frequency and distribution of spots, contained the recessive eye color vermilion (eosin singed vermilion). This was to test whether vermilion behaved in these sex-mosaics as Sturtevant ('20 *Proc. Soc. Exp. Biol. Med.*, **17**, 70–74), had observed that it did in ordinary gynandromorphs, viz., that it failed to show in the eye in which it was present, as known from other check characters. All other sex-linked characters had behaved as independent characters, uninfluenced by the constitution of the surrounding or bordering tissue. If vermilion failed to show in the piebald spots the eye-spot would show simply eosin (a pinkish yellow) but if vermilion was also independent then the eye spot should show the color of the double recessive form, viz., a pale cream. The tests showed that in the piebald spots vermilion failed to show, and the color was the same as though the father had been simply eosin.

The most important feature of the piebald individuals is that the elimination of the X-chromosome is caused in some way by a mutant change (Minute-n) carried in that chromosome. Moreover the effect of this change is not limited to the X, for in two cases all the maternal chromosomes were eliminated.

HAPLOIDY IN DROSOPHILA MELANOGASTER

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In a recent paper (*Amer. Nat.*, **1925**, page 128), a table was given in which the relation between sex and chromosomes in Drosophila melanogaster was shown for various combinations of chromosomes different from the usual diploid or 2*N* condition. Thus, 2N, 3N and 4N individuals are females. In conformity with the idea of genic balance, it was anticipated that the 1N individual would be female also, and this in spite of the apparent fact that in all known animal cases haploid individuals are males. The haploid was entered in the table as female, with a footnote to the effect

that the haploid type had not yet been found. Since then two mosaic individuals have occurred in each of which a section of the body showed characteristics indicating that the section was haploid and female.

Haploid No. 1, June 4, 1925.—In a cross (described in a preceeding paper, these PROCEEDIHGS, **11**, 701), between females heterozygous for the dominant sex-linked character Minute-n and males that carried the sex-linked recessives eosin eye-color and singed bristles and the third-chromosome recessive ebony body color, "piebald" individuals were produced. The piebalds showed spots that were not Minute-n but were eosin, singed and not-ebony. The spots were male, while the Minute-n individuals on which the spots occurred were females. Such piebald individuals were accounted for by an effect of the Minute-n gene upon the X-chromosome that carried it, of such a nature that this X frequently became lost from a cell in larval development. The patch of tissue descended from such a cell still had the paternal X and both the maternal and paternal sets of autosomes. The spots were accordingly male and patroclinous with respect to sex-linked characters and hybrid with respect to autosomal characters.

Besides the piebalds of the type just described, one individual occurred in the above cross that was different in several respects (Fig. 1). The spot was larger than that of any other head-spot seen, including all the left eye and the surrounding tissue of the left half of the head. The bristles of the spot were singed, and not Minute, though they were more delicate than normal bristles. The color of the included eye was eosin, but it differed from the eosin of all piebalds in that it was not the pinkish yellow characteristic of the male, but was dark, yellowish pink, the same color, as nearly as could be determined by comparison, as that of eosin females. The body-color of the spot was ebony, strikingly different from the remainder of the fly and differing in this respect from the piebald sibs, for in those there was no difference in body color between the spot and the surrounding tissue. Finally, the half of the head included in the spot was smaller than the right side, while no difference in size has been seen in the ordinary type of piebald. It was not only smaller, but the difference in size was relatively greater than that ever seen in gynandromorphs in which one half of the head is male and the other half female. Moreover,

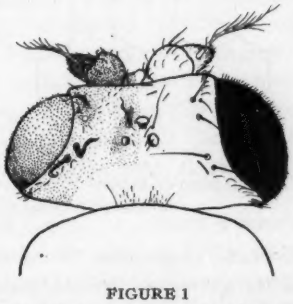

FIGURE 1

Haploid mosaic No. 1. Left side of head ebony in body color with singed bristles and with eosin eye color of the female type. Remainder of the female wild-type in body color with Minute-n bristles and normal red eye color.

the eye was disproportionately smaller than the rest of the left side of the head. The texture of the eye was noticeably finer than that of a normal eye, depending upon a smaller size of facet. The ocellus belonging to the left half was also disproportionately smaller.

The above characteristics are accountable on the assumption that in this case all the maternal chromosomes were eliminated, and the spot was strictly haploid and paternal. That it had one and only one X, and this from the father, is shown by the presence of the recessive sex-linked characters eosin and singed. That it had only one-third chromosome and this paternal was shown by the presence of the third-chromosome recessive character ebony. It was known from the breeding results with 3N mothers that zygotic combinations in which the second chromosome is not present in the same number as the third are lethal. While it is not certain that such an autosomal unbalance would prevent the development of a very small piece, it should at least cause a relatively large section, such as this, to be malformed, which was not the case. 3N and 4N females are regularly

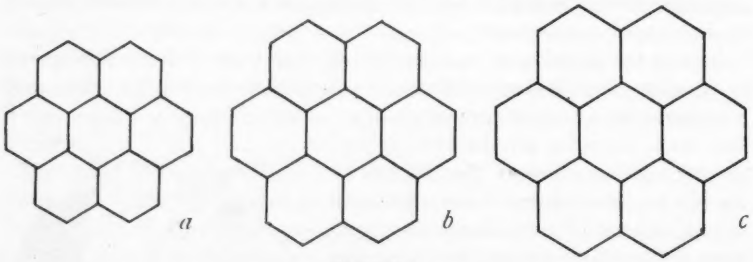

FIGURE 2

Facet diagram of haploid left eye of mosaic No. 1 (*a*), of diploid right eye (*b*), and of an unrelated triploid (*c*). Ratio ot diameters, 19:23:(26).

distinctly larger than 2N females, and conversely the small size of the parts on the left side of the head agree with expectation for haploidy. The eyes of 3N and 4N females are disproportionately larger than the rest of the head and form the easiest means of identification. Conversely the eye included in the spot was disproportionately smaller than the rest of the tissue and agrees with expectation for haploidy. Finally, the eye color of the spot was different from that characteristic of male tissue but was not different from that of the female. The female-eosin color of this spot tends to complete the proof that the spot is haploid and at the same time shows that in all probability a haploid individual, if it were able to live, would be female and not male. Analogy with the cases of 2N, 3N and 4N individuals, which are known to be females, would lead, through the idea of genic balance, to the expectation that haploid tissue would be female.

The mosaic fly was sectioned, and examined as to the size of nuclei

cells and other structures. Unfortunately, the plane of section did not pass through the two eyes symmetrically. Moreover, the nuclei and cells were too irregular and indefinite in outline and varied too much from tissue to tissue and with the angle of inspection for any accurate comparison to be made. A general impression was gained that the cells were slightly smaller in the left half of the head, especially in the eye, where they were more definite. The facets could be seen very distinctly, and measurements were made of the diameters of about 30 from the left eye and 15 from the right eye. Most of the facets of the left eye had a diameter of 19 units of the scale of the micrometer eye-piece. The range was small, from 18 to 20, with 20 almost unrepresented. The facets of the right eye had a mode of 23 units with a range of 21–23. This is a constant difference and strikingly great as can be seen from the hexagonal diagrams of figure 2*a* and 2*b*, where the diameters of the hexagons are in the ratio 19:23.

Figure 2*c* shows a diagram with a facet diameter of 26 units, corresponding to facets of the eye of a single triploid (3N) individual that was examined. The ommatidium of a fly is apparently a very definite structure in which a uniform number of cells are regularly arranged. Thus, the facet itself is secreted by two cells lying side by side beneath it. The crystalline cone is similarly formed by four cells beneath those, etc. Thus it is probable that the facet size is a very accurate index of the size of the cells composing the ommatidium. The cells of triploid individuals are readily seen to be larger than the cells of diploids, and correspondingly their facets are larger. The facet size of triploid females is the most definite character difference used in separating 3N from 2N females. From the criterion of facet size it is therefore probable that the cells of the supposedly haploid left eye were definitely smaller than those of the normal right eye.

Haploid No. 2, July 28, 1925.—In a cross between a Minute-n female and a yellow white forked[5] male, there appeared, in addition to the usual piebalds, one individual that differed from them in the following respects: the spot was the largest ever seen, covering the entire dorsal and ventral surfaces of the left half of the thorax, and extending from ventral mid-line to dorsal mid-line and from the tip of the scutellum to and including the humeral patch. The body color of the spot was yellow. The bristles were forked[5] and not Minute, but they were more delicate than the normal size bristles of the spots of piebalds, and were not much larger than the not-forked Minute-n bristles of the rest of the fly. The body-size of the spot was markedly smaller than that of the Minute-n part, while in piebalds the body-size of the spot, even in large spots, has not been distinguishable from that of the Minute-n tissue adjacent. The wing of the left side was yellow and bore forked bristles. It was only about three-fourths as large as the wing of the other side—a difference far greater than that seen

even between wings of the male and female side of a gynandromorph. All three left legs were included in the spot. These legs were only about four-fifths as long as the legs of the other side, and were even more slender in proportion. Especially noteworthy is the inclusion of the fore-leg, this being the only case among thousands of Minute-n females examined. The fore-leg bore no séx-comb. In the examination of gynandromorphs the sex-comb has been found an infallible index of the sex of the part. The absence of sex-comb was therefore evidence of the strongest kind that the spot was female and not male. The individual was lost after having been carefully described but before it could be drawn and sectioned.

The characteristics of the spot agreed with expectation for haploid tissue and with the chacteristics observed in the previous case that was more thoroughly checked by an autosomal recessive and by histological examination.

The unique feature of this case was the inclusion of the fore-leg, which made the femaleness certain. The evidence that haploidy is the correct explanation and that haploid tissue is female, is thus much stronger than it would have been from either case alone.

PLANT GROWTH

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To be of use to the physiologist, a formula should be an accurate expression of the process it describes, both quantitatively and with respect to the nature of that process. Robertson's[1] formula for growth describes it as an autocatalytic monomolecular reaction, but unfortunately the formula is not sufficiently elastic to fit the observed course of growth with adequate accuracy in all cases. In consequence Pearl and Reed[2] have suggested a modification that fits any growth curve with great accuracy, but which has no evident physiological significance, however valuable it may be for statistical purposes.

By purely empirical methods, it has been found possible to fit the following formula to any simple curve of plant growth:

$$y = A(1 - ae^{-K_1 t} + be^{-K_2 t} - ce^{-K_3 t} + \ldots\ldots) \qquad (1)$$

where y is the amount of growth, at time t, A the maximum amount of growth, K_1, K_2, K_3, etc., reaction constants and a, b, c, etc., constants to be determined. In this equation each successive exponential function is a corrective factor permitting accuracy of fit to any desired degree.

A simple form of Equation 1 is obtained when only two exponential functions are required. Equation 1 then becomes:

$$y = A(1 - ae^{-K_1 t} + be^{-K_2 t}) \tag{2}$$

If the growth curve starts at the origin, the values of a and b can be determined, since the rate of growth is zero when $t = 0$.

$$y = 0, t = 0 \quad \therefore \quad a - b = 1$$

$$\frac{dy}{dt} = 0, t = 0 \quad \therefore \quad aK_1 = bK_2$$

$$a = \frac{K_2}{K_2 - K_1}, \quad b = \frac{K_1}{K_2 - K_1}.$$

Equation 2 therefore becomes:

$$y = A(1 - \frac{K_2}{K_2 - K_1}e^{-K_1 t} + \frac{K_1}{K_2 - K_1}e^{-K_2 t}) \tag{3}$$

which is the formula for a consecutive monomolecular reaction of the type

$$A \longrightarrow x \longrightarrow y, \tag{4}$$

K_1 being the velocity constant of the reaction $x \longrightarrow y$ and K_2 the velocity constant of the reaction $A \longrightarrow x$.

Taken literally, this formula for growth implies that the maximum size of a plant is determined by the amount of substance available for the formation of the end products, that is, by the amount of nutrient material available for growth. Since this is not necessarily the case, another interpretation must be found before the proposed formula can be accepted.

If $(k_1 + h_1)$ be substituted for K_1, and $(k_2 + h_2)$ be substituted for K_2 in the above equation, it becomes the formula of a consecutive reversible monomolecular reaction of the type:

$$u \leftrightharpoons x \leftrightharpoons y, \tag{5}$$

k_1 being the velocity constant of the reaction $u \longrightarrow x$, h_1 the constant for the reverse reaction $x \longrightarrow u$, k_2 the constant for $x \longrightarrow y$, and h_2 the constant for $y \longrightarrow x$. A becomes the equilibrium value of y or $y\alpha$. There are many weighty reasons for considering the plant to be a system in equilibrium.[3]

On this basis plant growth may be considered a consecutive reversible monomolecular reaction of the type:

$$u \leftrightharpoons v \leftrightharpoons \ldots \leftrightharpoons w \leftrightharpoons x \leftrightharpoons y, \tag{6}$$

for which the formula would be:

$$y = y\alpha \left[1 - \sum_{i=1}^{n} \frac{\prod_{1}^{n}{}'(k_j + h_j)}{\left|\prod_{1}^{n}{}'(k_j + h_j - k_i - h_i)\right|} e^{-(k_i + h_i)t} \right], j \neq i \quad (7)$$

where n is the number of consecutive reversible reactions, k_i the velocity constants, $k_1, k_2, \ldots, k_n$, of the reactions leading to the formation of y and h_i the corresponding velocity constants of the reverse reactions.

In case the values $k_1 + h_1$, $k_2 + h_2$, $k_3 + h_3$, etc., can be arranged in an arithmetic series having the ratio 1:2:3:4, etc. Equation 7 becomes:

$$y = y\alpha(1 - e^{-Kt})^n \quad (8)$$

which is Mitscherlich's formula for growth, more commonly written in the logarithmic form:[4]

$$\log(\sqrt[n]{y\alpha} - \sqrt[n]{y}) = \log\sqrt[n]{y\alpha} - Kt. \quad (9)$$

This becomes the formula for a simple monomolecular reaction when $n = 1$ and Robertson's formula for growth when the y axis is moved to the point of inflection and $n = -1$.

Equation 8 has been found to give an excellent fit in many cases. Mitscherlich considers n to be the number of environmental factors affecting growth under given conditions. This interpretation is not accepted, because the formula describes a special class of consecutive monomolecular reactions.

If plant growth be a consecutive, reversible monomolecular reaction, as mathematical considerations derived from the shape of the growth curve suggest, there should be definite consequences deducible from this assumption and susceptible of experimental proof. It is characteristic of reversible reactions that an accumulation of any of the end products of the reaction retards its velocity, and in a consecutive reversible reaction the accumulation of end products would retard the velocities of the entire series of reactions from the beginning.

If the products of photosynthesis be considered end products of shoot growth in plants, the growth of shoots should be retarded by their accumulation. It is well known that as the rate of shoot growth decreases, carbohydrates accumulate. There has been insufficient evidence to warrant the assumption that the accumulation of carbohydrates is responsible for the diminishing growth rate, but since carbohydrates are the chief products of photosynthesis and may be considered end products of shoot growth, their accumulation should retard growth if it be a reversible reaction. It is noteworthy in this connection that carbohydrate accumulation does not retard root growth, but rather favors it.[5]

It has been observed that the rest period of plants—that period when buds and cambium are unable to grow no matter how favorable environmental conditions may be—is coincident with the period of maximum carbohydrate content.[6] Treatments that delay carbohydrate accumulation delay the beginning of the rest period and all the treatments known to be efficacious in breaking the rest period are likewise known to increase respiration and so accelerate carbohydrate consumption.[7] On the assumption that growth is a reversible reaction, the coincidence of the period of maximum carbohydrate content with the rest period becomes significant. It is also noteworthy that there is no rest period for root growth.

If growth be a consecutive reversible reaction, carbohydrate accumulation should retard or inhibit the initial stages of the reaction leading to carbohydrate manufacture. It has been established that flower bud formation in the apple is associated with carbohydrate accumulation in the vicinity of the bud at the time of differentiation.[8] A bud may become either a leaf bud or a flower bud. Leaf bud formation may be considered an early stage in a series of consecutive reactions that leads to carbohydrate manufacture, since it is an initial stage in the development of a leafy shoot. The fact that flower bud formation occurs following carbohydrate accumulation suggests that leaf bud formation has been inhibited. In apple spurs carbohydrate accumulates chiefly in the form of starch and this inert substance could hardly exert any material influence on bud differentiation. However, starch is unquestionably in equilibrium with other more active forms of carbohydrate and its accumulation is merely an easily detected index of general carbohydrate accumulation.

Murneek[9] has recently shown that fruiting in the tomato retards vegetative development and that the rate of growth increases when the fruit is removed. He suggests that some chemical compound may be produced in the developing fruit that stimulates fruit development at the expense of vegetative growth. If this posited substance be considered one of the end products of the vegetative growth of the plant as a whole, its formation and accumulation might be expected to retard vegetative development.

[1] Robertson, *Arch. Entwicklungsmech.*, **25**, 581 (1908).

[2] Pearl and Reed, *Proc. Nat. Acad. Sci.*, **8**, 365 (1922); *Metron*, **3**, 6 (1923).

[3] Cf. Hooker, *Amer. Nat.*, **53**, 506 (1919).

[4] Mitscherlich, *Landw. Jahrb.*, **53**, 167 (1919); *Fühling's Landw. Zeit.*, **68**, 130, 419 (1919).

[5] Cf. Starring, *Proc. Amer. Soc. Hort. Sci.*, **20**, 288 (1923).

[6] Abbott, *Bot. Gaz.*, **76**, 167 (1923).

[7] Cf. Rosa, *Proc. Amer. Soc. Hort. Sci.*, **20**, 180 (1923).

[8] Hooker, *Mo. Agric. Exp. Sta. Research Bul.*, **40** (1920).

[9] Murneek, *Bot. Gaz.*, **79**, 329 (1925).